R-OSSE
Acoustic Waveguide

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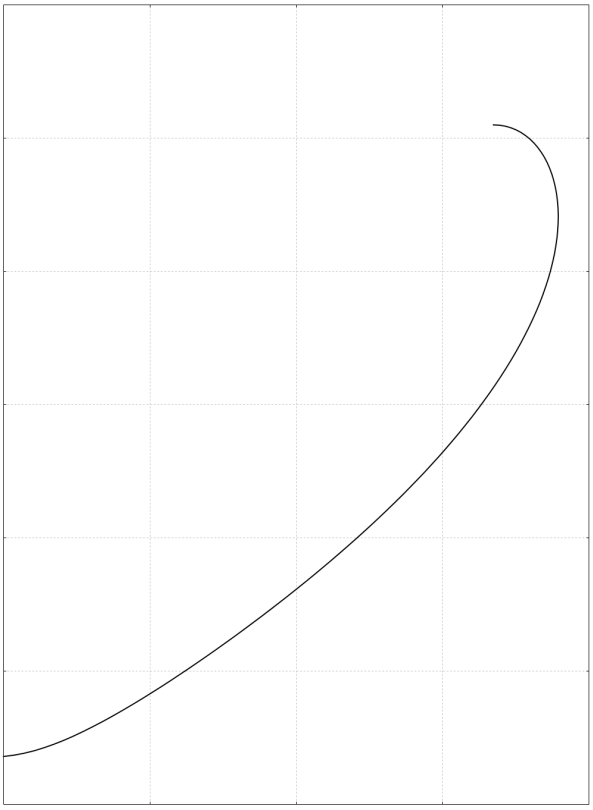
April 2022, rev.4



Introduction

In 2019 the OSSE (or "OS-SE") waveguide formula was presented[1], extending the well-known Oblate Spheroidal (OS) waveguide by incorporating a smooth termination into a flat panel. While it proved the importance of the added gradual termination, due to its inherent half-space nature the usefulness was still somewhat limited - for a real-life use it is necessary to place such device into a finite baffle with an additional edge treatment which is not any more a part of its analytical description.

The now presented R-OSSE set of parametric equations goes a step further and defines a complete waveguide terminated into a free space by means of a convenient, self-containing analytical description. Such approach can be readily used e.g. in further optimization algorithms, CAD routines, etc.

The R-OSSE parametric description

In the following text we describe a shape of a profile of an axisymmetric waveguide as a set of coordinates [x, y], where 'x' denotes the axial distance from the throat and 'y' the distance of the profile point from the axis.

Because the OSSE has the form of a function y(x), it can't describe shapes that fold back as the profile curve progresses. For this we need a parametric description in a form [x, y] = [x(t), y(t)], where x(t) and y(t) are some functions of a new parameter 't'. Typically these functions are constructed so that the parameter 't' ranges from 0 to 1.

The functions used in the R-OSSE description are plotted on the Fig 1.

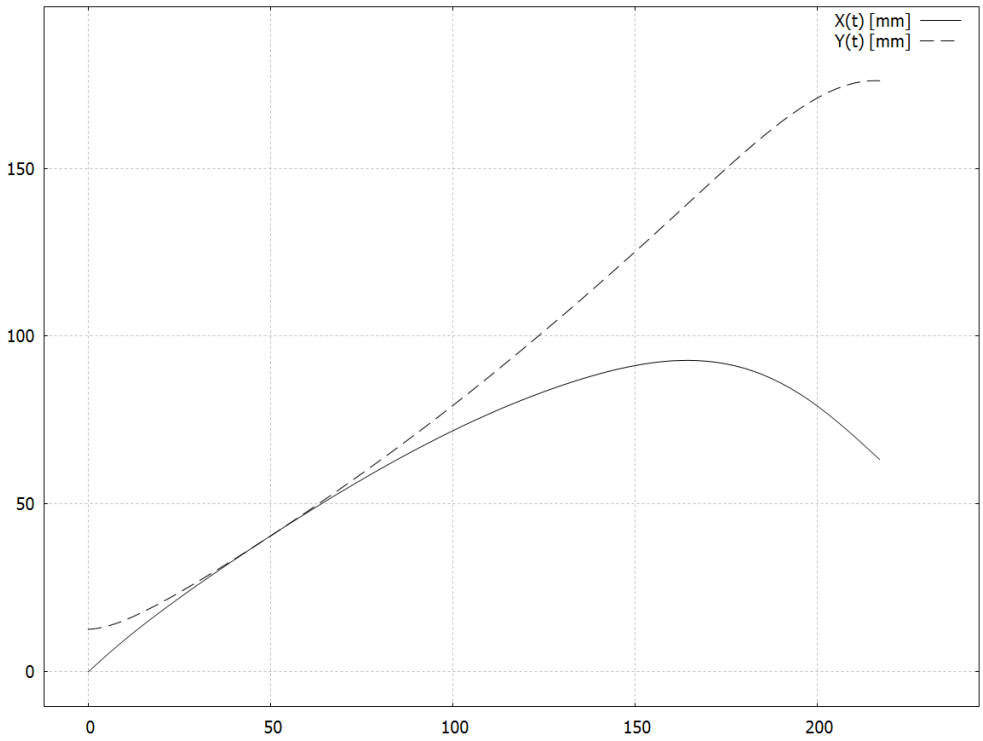


Fig. 1: R-OSSE x(t), y(t) components

1 http://www.at-horns.eu/release/OS-SE Waveguide.pdf

The functions on Fig. 1 are constructed by means of two conic sections each. The function x(t) is simply a difference of x1(t) and x2(t), a hyperbola and a parabola (Fig. 2). The function y(t) is a weighted average of y1(t) and y2(t), both being hyperbolas, starting as y2 and ending as y1 (Fig. 3).

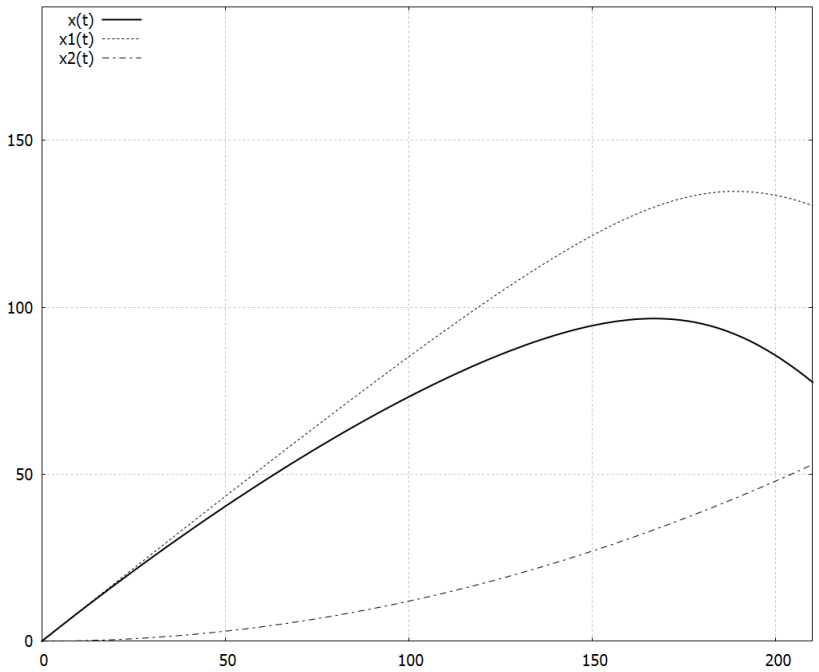


Fig. 2: x(t) components

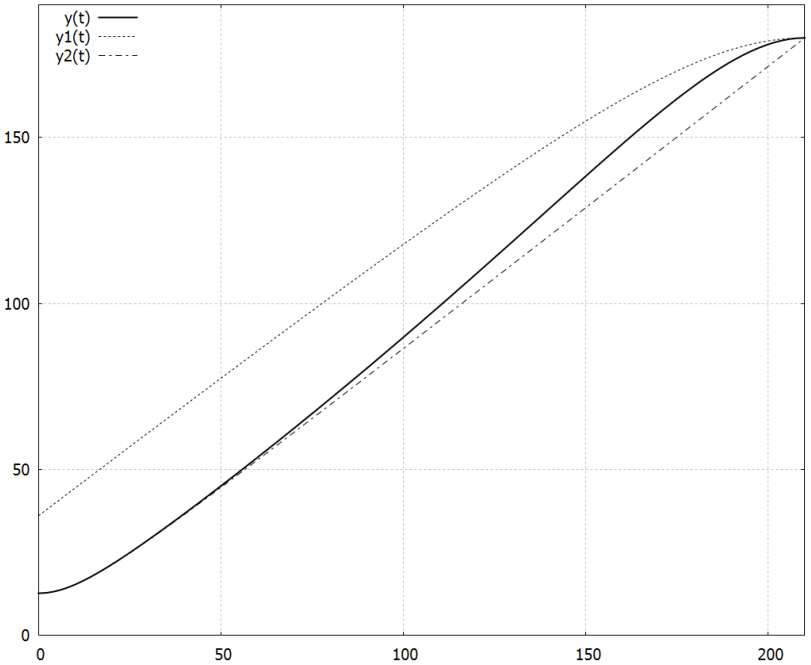


Fig. 3: y(t) components

R-OSSE design formulae

Design Parameter	Description	unit/example
R	Waveguide outer radius	[mm]/190
a	Nominal coverage angle	[deg]/40
r_0	Throat radius	[mm]/18
a_0	Throat opening angle	[deg]/0
k	Throat expansion factor	1
r	Apex radius factor	0.3
m	Apex shift factor	0.8
b	Bending factor	0.3
q	Throat shape factor	3

Auxiliary constants

$$k_1 = (kr_0)^2$$

$$k_2 = 2r_0 \tan(a_0)$$

$$k_3 = \tan^2(a)$$

$$L = \frac{\sqrt{k_2^2 - 4k_3(k_1 - (R + r_0(k-1))^2)} - k_2}{2k_3}$$

Core functions

$$x_1(t) = \sqrt{r^2 + m^2} - \sqrt{r^2 + (t-m)^2}$$

$$x_2(t) = bt^2(\sqrt{r^2 + m^2} - \sqrt{r^2 + (1-m)^2})$$

$$y_1(t) = L(1 - \sqrt{1 + k_3(t-1)^2}) + y_2(1)$$

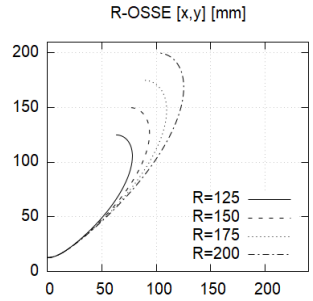
$$y_2(t) = \sqrt{k_1 + k_2 Lt + k_3 L^2 t^2} - r_0(k-1)$$

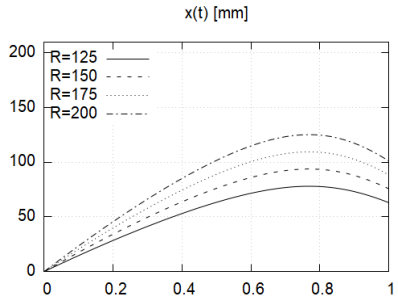
R-OSSE parametric equation

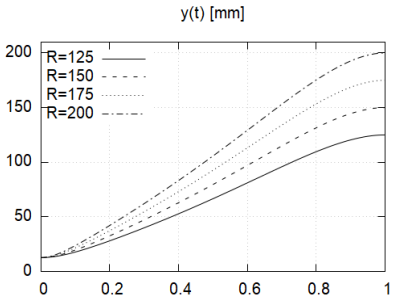
$$x(t) = L(x_1(t) - x_2(t))$$

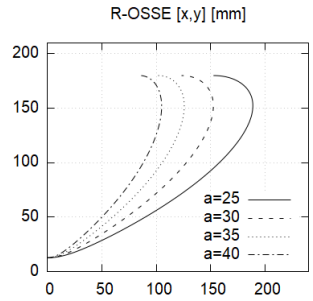
$$y(t) = t^q y_1(t) + (1 - t^q) y_2(t) \quad , \quad t \in \langle 0,1 \rangle$$

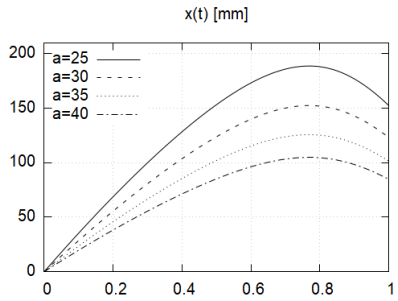
The following charts give an overview of the effect of each individual design parameter on the resulting shape.

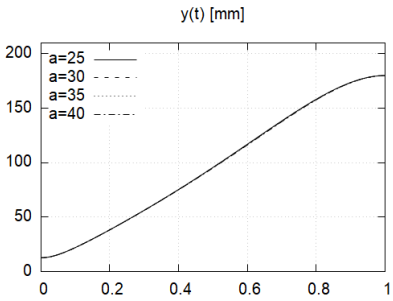


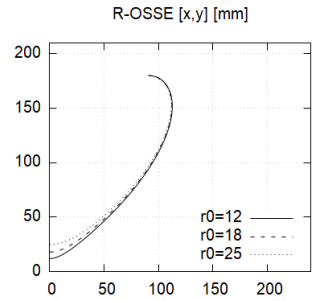


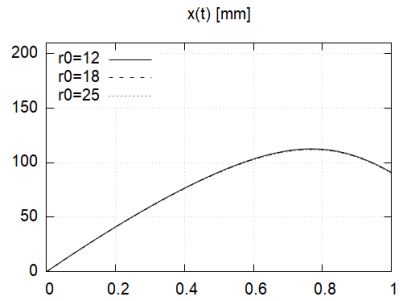


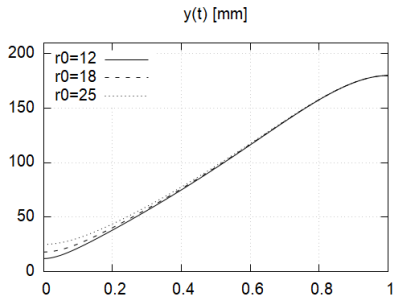


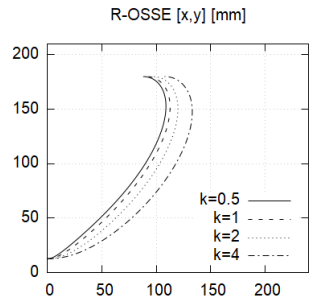


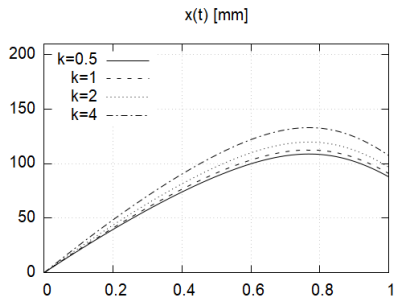


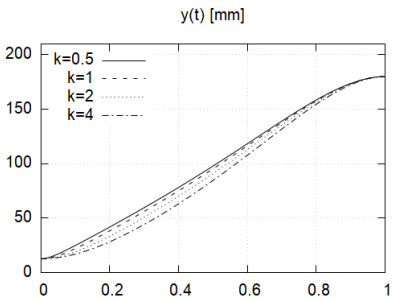


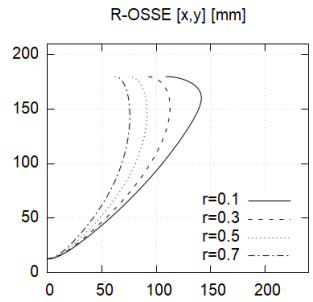








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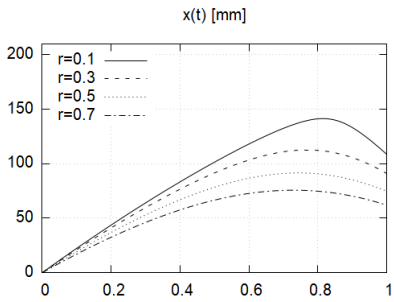
r=0.3
r=0.5
r=0.7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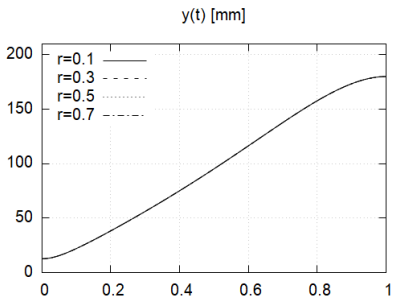
r=0.1
r=0.3
r=0.5
r=0.7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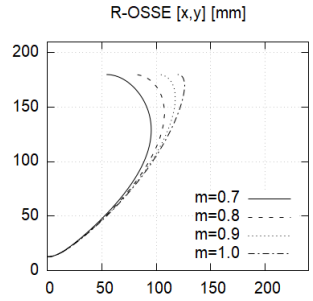
r=0.3
r=0.5
r=0.7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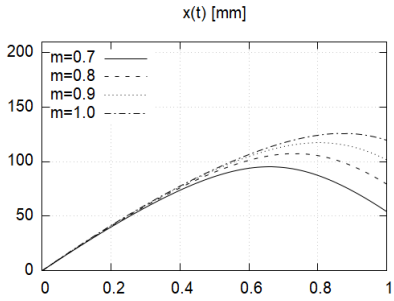
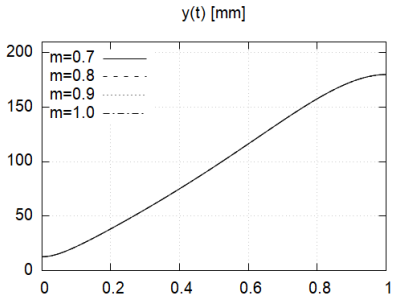
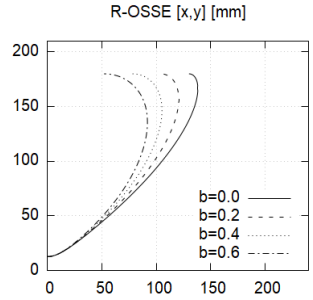
m=0.8
m=0.9
m=1.0


x(t) [mm]
m=0.7
m=0.8
m=0.9
m=1.0


y(t) [mm]
m=0.7
m=0.8
m=0.9
m=1.0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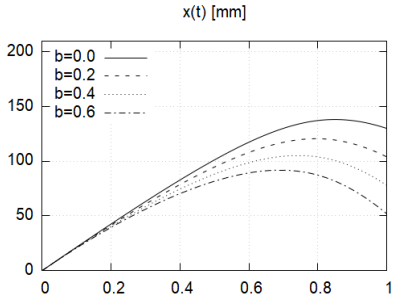
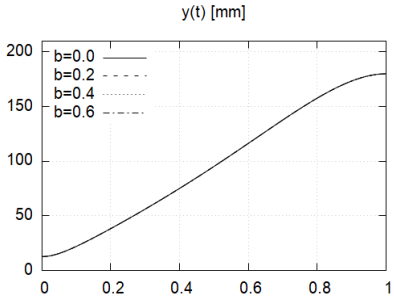
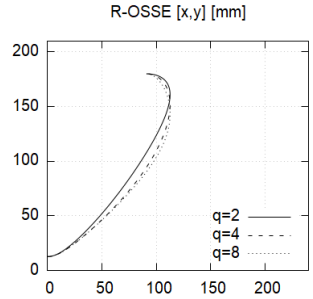
b=0.2
b=0.4
b=0.6


x(t) [mm]
b=0.0
b=0.2
b=0.4
b=0.6


y(t) [mm]
b=0.0
b=0.2
b=0.4
b=0.6


R-OSSE [x,y] [mm]
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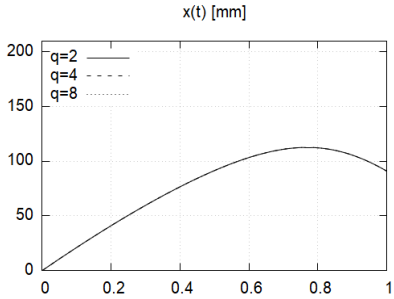
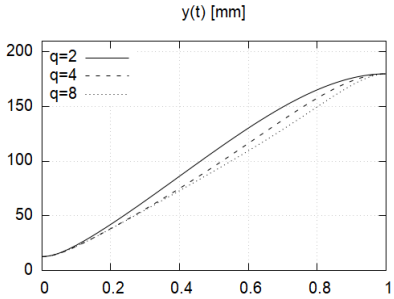
q=4
q=8


x(t) [mm]
q=2
q=4
q=8


y(t) [mm]
q=2
q=4
q=8

Design example

R-OSSE free standing waveguide, ⌀570 x 257 mm, 1.4" throat (BEM simulation):